WORK IT OUT

A maths workbook — concepts, worked examples, graded practice and number puzzles

Do the Learn First pages before the practice. Do the practice before the puzzles. Full solutions are at the back — but they only help you after you have tried.

Part	What it is	How to use it	Time
1	Learn First	Read and annotate. Copy every worked example by hand.	25 min
2	Worked Examples	Cover the solution. Try. Then compare line by line.	20 min
3	Guided Practice	Steps are given; you fill the gaps.	15 min
4	Practice Set A · Fluency	Straight application. Aim for speed and accuracy.	20 min
5	Practice Set B · Reasoning	Multi-step. Show every line.	25 min
6	Practice Set C · Challenge	Stretch problems. One may beat you — that is fine.	25 min
7	The Crossnumber	Answers are numbers. Wrong maths will not fit.	20 min
8	Number Search, Magic Square & Concept Bingo	Solo or with a partner.	20 min
9	Spot the Mistake	Audit a worked solution. Find the planted error.	15 min
10	Quick-Fire Round	Matching, True/False, Two Truths and a Lie.	15 min
11	The Escape Challenge	Teams of 4.	25 min

Part 1 · Learn First

A test tube is not a cylinder. A playing top is not a cone. Real objects are almost always combinations of the basic solids — and the whole trick of this chapter is knowing exactly which surfaces survive when two solids are glued together.

Surface area formulas for the basic solids

These are the building blocks. Every combination problem in this chapter reduces to picking the right pieces from this table.

Slant height of a cone: $l=\sqrt{r^2+h^2}$, where h is the vertical height and r the base radius.

Frustum of a cone (a cone with its top sliced off, radii $r_1>r_2$, slant height l): curved surface area ¿$\pi(r_1+r_2)l$, total surface area ¿$\pi(r_1+r_2)l+\pi r_1^2+\pi r_2^2$. This shape is included here for reference; it does not appear in the current edition's exercises.

Solid	Curved/Lateral surface area	Total surface area
Cuboid (l, b, h)	2h(l+b)	2(lb+bh+hl)
Cube (edge a)	4a²	6a²
Cylinder (r, h)	2πrh	2πr(h+r)
Cone (r, l = slant height)	πrl	πr(l+r)
Sphere (r)	4πr²	4πr²
Hemisphere (r)	2πr²	3πr²

Rule to remember

- CSA (curved surface area) counts only the curved part; TSA (total surface area) adds the flat circular base(s) too.
- A hemisphere has **two** surface areas worth knowing: its curved part $2\pi r^2$ and its flat circular base πr^2 — you use whichever one is actually exposed.

Surface area of a combination of solids

Whenever two solids are joined, some surface disappears at the joint — it is now hidden inside the new solid, not part of its outer skin.

Golden rule: the total surface area of a combined solid is the sum of the *curved* (or otherwise exposed) surface areas of its parts — **never** simply the sum of the individual total surface areas, because the flat faces where the pieces meet are covered up and stop being part of the outer surface.

Example pattern (cone + hemisphere, e.g. a playing top):
$TSA\ of\ toy = CSA\ of\ hemisphere + CSA\ of\ cone$ — both flat circular faces vanished into the join.

Example pattern (cube + hemisphere, e.g. a decorative block):
$Surface\ area = TSA\ of\ cube - base\ area\ of\ hemisphere + CSA\ of\ hemisphere$ — here we must *subtract* the circle of cube-face that the hemisphere covers, then add back only the hemisphere's curved part.

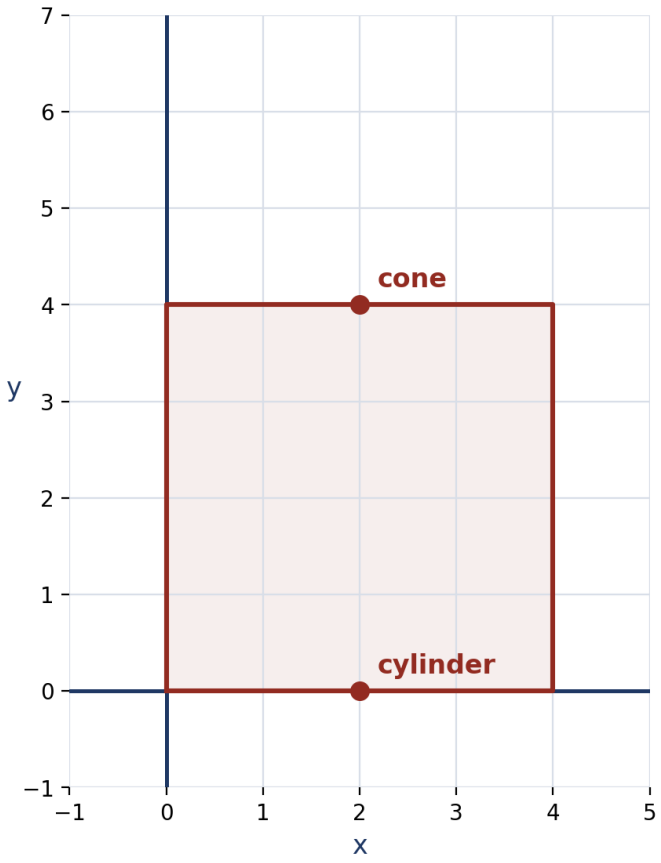

Fig. 12.A — a cylinder with a cone on top: only the curved surfaces and one flat circular base survive as the outer skin

Painting a rocket (cone on cylinder, unequal radii)

A toy rocket is a cone mounted on a cylinder. Total height 26 cm, conical part height 6 cm, cone base diameter 5 cm, cylinder base diameter 3 cm. Take $\pi = 3.14$. Find the area painted orange (cone) and yellow (cylinder).

Cone: $r = 2.5$ cm, $h = 6$ cm, so $l = \sqrt{r^2 + h^2} = \sqrt{2.5^2 + 6^2} = 6.5$ cm.

Cylinder: $r = 1.5$ cm, $h = 26 - 6 = 20$ cm.

Because the cone's base (radius 2.5) is wider than the cylinder's top (radius 1.5), a ring of the cone's base is exposed too:

Orange area

¿ $\pi r l + \pi r^2_{cone} - \pi r^2_{cyl} = 3.14[(2.5)(6.5) + 2.5^2 - 1.5^2] = 3.14 \times 20.25 = 63.585\, cm^2$

Yellow area ¿ $2\pi r h + \pi r^2 = \pi r(2h + r) = 3.14 \times 1.5 \times 41.5 = 195.465\, cm^2$

Volume of a combination of solids

Volume behaves much more simply than surface area — nothing is hidden or lost when two solids are glued together, so volumes just add.

Golden rule: the volume of a combined solid **is** the sum of the volumes of its parts — unlike surface area, no adjustment is needed, because the material inside each piece is still all there.

When a solid is *scooped out* of another (a depression, a cavity, lead shots displacing water), subtract the volume removed instead of adding it.

Volume formulas for the basic solids

- Cuboid: $V = l\,b\,h$
- Cube: $V = a^3$
- Cylinder: $V = \pi r^2 h$
- Cone: $V = \frac{1}{3}\pi r^2 h$
- Sphere: $V = \frac{4}{3}\pi r^3$
- Hemisphere: $V = \frac{2}{3}\pi r^3$
- Frustum (radii r_1, r_2, height h): $V = \frac{1}{3}\pi h(r_1^2 + r_1 r_2 + r_2^2)$ (reference only)

A juice glass with a hemispherical base (apparent vs actual capacity)

A glass is a cylinder of inner diameter 5 cm and height 10 cm, but its bottom has a raised hemisphere of the same radius. Take $\pi = 3.14$. Find the apparent and actual capacity.

Apparent capacity (as if fully cylindrical) ¿$\pi r^2 h = 3.14$¿

Volume of the raised hemisphere (which the water cannot occupy)

¿$\frac{2}{3}\pi r^3 = \frac{2}{3}(3.14)$¿

Actual capacity ¿$196.25 - 32.71 = 163.54\ cm^3$

Lesson: "capacity of a container" always means volume actually available, not the volume you would compute by ignoring the obstruction.

Working with a solid inside another solid

Some problems circumscribe or inscribe one combined solid inside a simple one — the volume "left over" is found by subtraction, exactly as with cavities.

Pattern: if a right circular cylinder just encloses a hemisphere-plus-cone toy (same radius, height = radius of hemisphere + height of cone), then $\textit{difference of volumes} = \textit{volume of cylinder} - \textit{volume of toy}$.

Pattern: if a solid cone-on-hemisphere is lowered into a cylinder full of water, the water that overflows (or is "left" if not full) equals $\textit{volume of cylinder} \pm \textit{volume of solid}$, chosen to match the physical situation described.

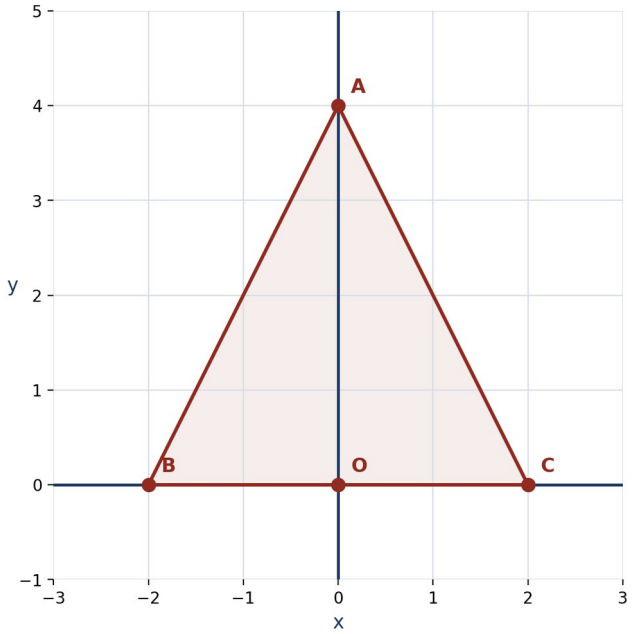

Fig. 12.B — a cone standing on a hemisphere of the same radius, the combination used in Example 7

A cone-on-hemisphere toy inside its circumscribing cylinder

A solid toy is a hemisphere surmounted by a cone. Cone height 2 cm, base diameter 4 cm. Take $\pi = 3.14$. Find the volume of the toy, and the difference between the volumes of the toy and the smallest cylinder that contains it.

Radius $r = 2$ cm (shared by cone and hemisphere).

Volume of toy

$$\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = \frac{2}{3}(3.14)(8) + \frac{1}{3}(3.14)(4)(2) = 16.75 + 8.37 = 25.12\ cm^3$$

Circumscribing cylinder: radius 2 cm, height $2 + 2 = 4$ cm (hemisphere radius + cone height).

Volume of cylinder $\pi r^2 h = 3.14(4)(4) = 50.24\ cm^3$

> Difference ¿$50.24 - 25.12 = 25.12\,cm^3$ — exactly equal to the volume of the toy itself, a coincidence worth noticing but not a general rule.

Part 2 · Worked Examples

Cover the right-hand column with a sheet of paper. Try the problem yourself first. Then uncover it one line at a time and check where your reasoning parted from the model.

Example 1 — A playing top (lattu): cone surmounted by a hemisphere

A top is shaped like a cone surmounted by a hemisphere. Total height 5 cm, diameter 3.5 cm. Take $\pi = \frac{22}{7}$. Find the area to be coloured.

Radius $r = 1.75$ cm (same for hemisphere and cone).

Step 1. CSA of hemisphere ¿ $2\pi r^2 = 2 \times \frac{22}{7} \times 1.75^2 \approx 19.25\ cm^2$.

Step 2. Height of cone ¿ $5 - 1.75 = 3.25$ cm. Slant height $l = \sqrt{r^2 + h^2} = \sqrt{1.75^2 + 3.25^2} \approx 3.7$ cm.

Step 3. CSA of cone ¿ $\pi r l = \frac{22}{7} \times 1.75 \times 3.7 \approx 20.35\ cm^2$.

Step 4. Total area to colour ¿ $19.25 + 20.35 \approx 39.6\ cm^2$.

> **Why this works**
>
> Note carefully: this is NOT the sum of the two solids' total surface areas — the flat circular faces where they join are never counted, because they are not part of the outer skin of the top.

Example 2 — A decorative block: cube with a hemisphere on top

A cube of edge 5 cm has a hemisphere of diameter 4.2 cm fixed on top. Take $\pi = \frac{22}{7}$. Find the total surface area of the block.

Step 1. TSA of cube ¿ $6a^2 = 6 \times 5^2 = 150\ cm^2$.

Step 2. Radius of hemisphere $r = 2.1$ cm. Base area of hemisphere it covers ¿ $\pi r^2 = \frac{22}{7} \times 2.1^2 = 13.86\ cm^2$.

Step 3. CSA of hemisphere ¿ $2\pi r^2 = 27.72\ cm^2$.

Step 4. Surface area of block ¿ $150 - 13.86 + 27.72 = 163.86\ cm^2$.

> **Why this works**
>
> The circle where the hemisphere sits is subtracted once (it is no longer cube surface) and the hemisphere's curved area is added once — a net *addition* of πr^2 compared to the plain cube, since $2\pi r^2 - \pi r^2 = \pi r^2$.

Example 3 — A bird-bath: cylinder with a hemispherical depression

A cylindrical bird-bath, height 1.45 m, radius 30 cm, has a hemispherical depression cut into one end. Take $\pi = \frac{22}{7}$. Find the total surface area.

Convert to the same unit: $h = 145$ cm, $r = 30$ cm.

Step 1. Because the hemisphere is a *depression* (scooped in, not stuck on), its curved surface still simply adds to the cylinder's curved surface — the flat circular rim of the depression coincides with the top circle of the cylinder, which was never separately counted.

TSA $= CSA\ of\ cylinder + CSA\ of\ hemisphere = 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

Step 2. $= 2 \times \frac{22}{7} \times 30 \times (145 + 30) = 2 \times \frac{22}{7} \times 30 \times 175 = 33000\ cm^2 = 3.3\ m^2$.

> **Why this works**
>
> A depression and a bump use the *same* formula pattern here only because the bird-bath has no separate flat base of its own being covered — always check what surface is actually exposed before reusing a memorised formula.

Example 4 — A shed: cuboid surmounted by a half-cylinder (volume)

A shed is a cuboid, base $15\ m \times 7\ m$, cuboidal height 8 m, surmounted by a half-cylinder of diameter 7 m and length 15 m. Take $\pi = \frac{22}{7}$. Find the volume of air, then subtract machinery (300 m³) and 20 workers at 0.08 m³ each.

Step 1. Volume of cuboid $= 15 \times 7 \times 8 = 840\ m^3$.

Step 2. Volume of half-cylinder $= \frac{1}{2}\pi r^2 h = \frac{1}{2} \times \frac{22}{7} \times 3.5^2 \times 15 = 288.75\ m^3$.

Step 3. Total volume $= 840 + 288.75 = 1128.75\ m^3$.

Step 4. Space occupied $= 300 + 20(0.08) = 301.6\ m^3$.

Step 5. Air available $= 1128.75 - 301.6 = 827.15\ m^3$.

> **Why this works**
>
> Volume problems never need the "subtract the joint" correction that surface-area problems do — the two volumes simply add, because no material actually disappears when solids merge.

Part 3 · Guided Practice

The method is laid out for you. Fill in the blanks. If you can do these without looking back at Part 2, you are ready for Part 4.

1. A vessel is a hollow hemisphere (diameter 14 cm) mounted on a hollow cylinder, total height 13 cm. Take $\pi = \frac{22}{7}$. Find the inner surface area.

Radius $r =$ ¿ _____ cm.

Height of cylindrical part ¿ $13 - r =$ ¿ _____ cm.

Inner surface area ¿ $2\pi rh + 2\pi r^2 = 2\pi r(h + r) =$ ¿ _____ cm^2.

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2. A cubical block of side 7 cm is surmounted by the largest possible hemisphere. Take $\pi = \frac{22}{7}$. Find the surface area of the solid.

Greatest possible diameter of the hemisphere ¿ _____ cm, so radius ¿ _____ cm.

Surface area ¿ $TSA\ of\ cube - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2 =$ ¿ _____ + ¿ _____ .

Total surface area ¿ _____ cm^2.

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3. A solid cone (height 120 cm, radius 60 cm) sits on a hemisphere of radius 60 cm inside a cylinder of radius 60 cm and height 180 cm, full of water. Find the volume of water left.

Volume of cylinder ¿ $\pi r^2 h =$ ¿ _____ cm^3.

Volume of cone ¿ $\frac{1}{3}\pi r^2 h =$ ¿ _____, volume of hemisphere ¿ $\frac{2}{3}\pi r^3 =$ ¿ _____ .

Volume of water left ¿ cylinder − ¿ (cone + ¿ hemisphere) ¿ _____ cm^3.

Practice Set A · Fluency

Straight application of the method. Aim to get every one right, then aim to get them right quickly.

#	Question	Your working and answer
1	2 cubes, each of volume 64 cm³, are joined end to end. Find the surface area of the resulting cuboid.	
2	A toy is a cone of radius 3.5 cm mounted on a hemisphere of the same radius. The total height of the toy is 15.5 cm. Find the total surface area. (Take $\pi = \frac{22}{7}$.)	
3	A hemispherical depression is cut from one face of a cubical wooden block such that the diameter of the hemisphere equals the edge of the cube. Write the expression (in terms of the edge a) for the surface area of the remaining solid.	
4	A medicine capsule is a cylinder with two hemispheres stuck to each end. The entire capsule is 14 mm long and 5 mm in diameter. Find its surface area. (Take $\pi = \frac{22}{7}$.)	
5	A solid is a cone standing on a hemisphere, both of radius 1 cm, and the height of the cone equals its radius. Find the volume of the solid in terms of π.	
6	From a solid cylinder of height 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm². (Take $\pi = \frac{22}{7}$.)	

#	Question	Your working and answer
7	A wooden article is made by scooping out a hemisphere from each end of a solid cylinder of height 10 cm and base radius 3.5 cm. Find its total surface area. (Take $\pi = \frac{22}{7}$.)	

Practice Set B · Reasoning

Two or more steps, or a step you have to choose for yourself. Show every line — marks live in the working.

#	Question	Your working and answer
1	A tent is a cylinder (height 2.1 m, diameter 4 m) surmounted by a cone of slant height 2.8 m. Find the area of canvas used, and its cost at ₹500 per m². (Take $\pi = \frac{22}{7}$; the base is not covered.)	
2	Rachel makes a model shaped like a cylinder with two cones attached at its ends. Diameter 3 cm, total length 12 cm, each cone of height 2 cm. Find the volume of air enclosed. (Take $\pi = \frac{22}{7}$.)	
3	A gulab jamun is a cylinder with two hemispherical ends, total length 5 cm, diameter 2.8 cm, and contains syrup up to 30% of its volume. Find approximately how much syrup is in 45 such gulab jamuns. (Take $\pi = \frac{22}{7}$.)	
4	A pen stand (cuboid $15\,cm \times 10\,cm \times 3.5\,cm$) has 4 conical depressions of radius 0.5 cm and depth 1.4 cm. Find the volume of wood in the stand. (Take $\pi = \frac{22}{7}$.)	
5	A solid iron pole is a cylinder of height 220 cm, base diameter 24 cm, surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given 1 cm³ of iron has mass 8 g. (Take $\pi = 3.14$.)	

#	Question	Your working and answer
6	A spherical glass vessel has a cylindrical neck 8 cm long and 2 cm in diameter; the diameter of the spherical part is 8.5 cm. A child measures its volume as 345 cm^3. Check whether this is (approximately) correct. (Take $\pi = 3.14$.)	

Practice Set C · Challenge

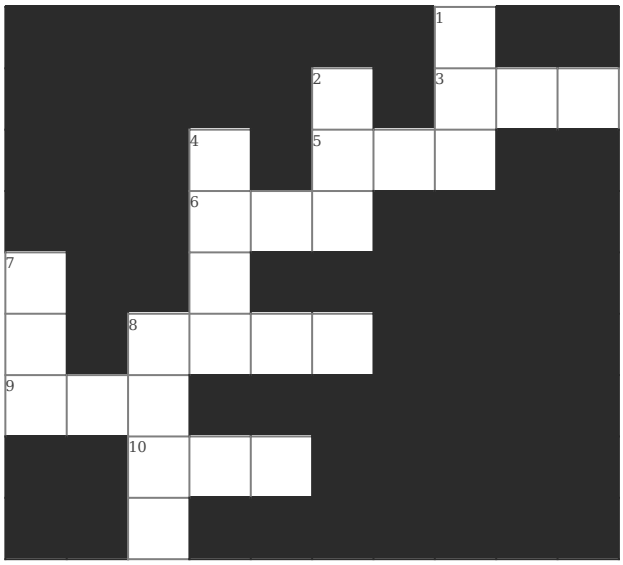
Harder, and deliberately so. If one defeats you, write down exactly where you got stuck. That sentence is worth more than a copied answer.

#	Question	Your working and answer
1	An inverted conical vessel of height 8 cm and top radius 5 cm is filled with water to the brim. Lead shots, each a sphere of radius 0.5 cm, are dropped in until one-fourth of the water flows out. Find the number of lead shots dropped.	
2	A juice glass has inner diameter 5 cm and height 10 cm, with a hemispherical raised base of the same radius. Find the apparent capacity and the actual capacity of the glass, and the percentage of apparent capacity that is "lost" to the raised base. (Take $\pi = 3.14$.)	
3	A solid toy is a hemisphere of radius 3 cm surmounted by a cone of height 4 cm and the same base radius. A cylinder of the same radius circumscribes the toy exactly. Find the volume of the toy and the difference between the volumes of the cylinder and the toy. (Take $\pi = 3.14$.)	
4	Explain, with reference to the playing-top example, why "total surface area of a combined solid" is never simply the sum of the total surface areas of its separate parts. Illustrate with one concrete numerical example of your own choosing.	

#	Question	Your working and answer
5	A cuboidal shed of base $15\,m \times 7\,m$ and cuboidal height 8 m is surmounted by a half-cylinder of diameter 7 m and length 15 m, exactly as in the industrial shed example. If a second, identical shed is built, but this time the half-cylinder is replaced by a full hemisphere-topped cylinder of the same base circle, would the enclosed volume of air increase or decrease, and by how much? (Take $\pi = \frac{22}{7}$.)	

Part 7 · The Crossnumber

Every answer is a number. One digit per square. Because answers cross, a wrong calculation simply will not fit — this puzzle marks itself.



ACROSS

3. TSA of a cube of edge 7 cm, in cm² *(3 digits)*

5. TSA of a cube of edge 5 cm, in cm² *(3 digits)*

6. CSA of a cylinder, r=7 cm, h=12 cm, in cm² (π=22/7) *(3 digits)*

8. TSA of a hemisphere of radius 7 cm, in cm² (π=22/7) *(4 digits)*

9. CSA of a cone, r=7 cm, l=21 cm, in cm² (π=22/7) *(3 digits)*

10. Volume of a cube of edge 7 cm, in cm³ *(3 digits)*

DOWN

1. CSA of a cone, r=7 cm, l=10 cm, in cm² (π=22/7) *(3 digits)*

2. TSA of a cylinder, r=7 cm, h=9 cm, in cm² (π=22/7), to nearest whole number *(3 digits)*

4. Volume of a sphere of radius 7 cm, in cm³, to nearest whole number (π=22/7) *(4 digits)*

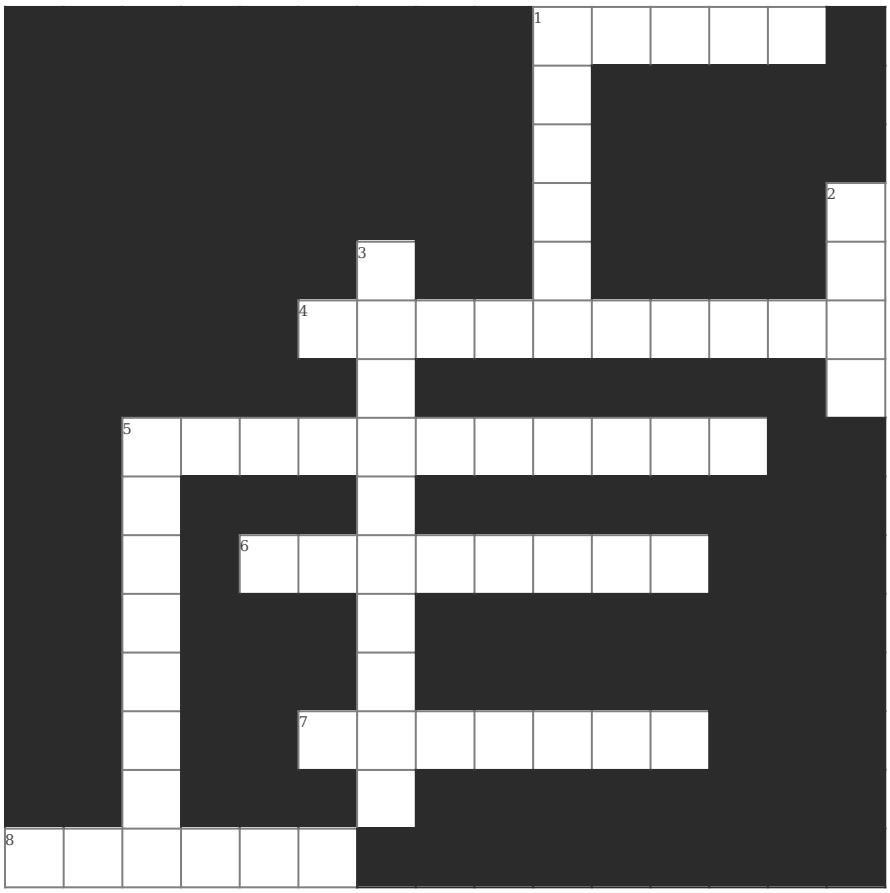
7. CSA of a hemisphere of radius 7 cm, in cm² (π=22/7) *(3 digits)*

8. CSA of a sphere of radius 14 cm, minus 384, in cm² (π=22/7) *(4 digits)*

Bonus questions that did not fit the grid — work them out anyway: TSA of a sphere of radius 7 cm, in cm² (π=22/7)

Part 7b · The Crossword

Same rules as an ordinary crossword — but every word is maths vocabulary from this chapter.



ACROSS

1. ___ height, the distance from the apex of a cone to its base circle (5)

4. a hollow scooped out of a solid, e.g. in a bird-bath (10)

5. a solid made by joining two or more basic solids together (11)

6. the volume a hollow container can actually hold (8)

7. what remains of a cone after its top is sliced off parallel to the base (7)

8. the type of surface area that excludes flat circular faces (6)

DOWN

1. a perfectly round solid, every surface point equidistant from the centre (6)

2. a solid that tapers smoothly from a circular base to a single apex (4)

3. exactly half a sphere, cut through its centre (10)

5. a solid with two parallel circular bases joined by a curved surface (8)

Part 8 · Number Search & Magic Square

Work out each answer below, then find it hidden in the digit grid — across, down, diagonally or backwards.

1. CSA of a cylinder, r=7, h=12 (π=22/7)
2. TSA of a sphere, r=7 (π=22/7)
3. Volume of a sphere, r=7, nearest whole (π=22/7)
4. CSA of a hemisphere, r=7 (π=22/7)
5. TSA of a cube, edge 7
6. Volume of a cube, edge 7
7. TSA of a hemisphere, r=7 (π=22/7)
8. CSA of a cone, r=7, l=21 (π=22/7)
9. CSA of a cone, r=7, l=10 (π=22/7)
10. TSA of a cube, edge 5

5	5	7	0	4	4	9	6	3	9	4	3	2
6	9	0	9	9	9	3	0	2	3	2	2	1
1	5	8	7	7	4	3	5	9	2	5	6	2
6	6	7	8	6	5	6	7	0	5	8	4	8
1	1	0	0	7	1	3	8	5	5	5	2	8
2	6	1	1	4	4	1	6	5	7	4	9	5
0	6	0	5	1	1	7	4	6	5	6	3	9
2	2	0	8	1	0	6	5	1	6	7	5	2
0	2	0	2	5	4	1	4	9	3	4	0	2
2	9	4	1	4	7	8	4	6	5	4	8	4
3	4	3	2	0	8	6	8	4	7	2	5	6
7	8	0	3	5	0	6	2	6	5	9	2	6
6	5	2	9	6	2	9	4	8	8	8	5	9

☐ 1540	☐ 1078	☐ 528	☐ 616	☐ 154
☐ 294	☐ 343	☐ 462	☐ 220	☐ 150

Magic Square — every row, column and diagonal adds to the same total.

A 3×3 grid using the consecutive integers 1–9, one per box, every row, column and diagonal summing to the same magic constant.

8	**1**	**6**
3		**7**

Magic constant: 15. Fill the empty cells.

Part 9 · Spot the Mistake

Each solution below reaches a wrong answer, and exactly one step is to blame. Find the first wrong step and rewrite it correctly. Marking other people's work is the fastest way to stop making their mistakes.

🔍 Spot the Mistake

Surface area of a playing top (cone + hemisphere)

Step 1. The top is a cone surmounted by a hemisphere sharing the same radius.

Step 2. Total surface area = TSA of cone + TSA of hemisphere.

Step 3. So we add $\pi r(l+r)$ and $3\pi r^2$.

_____ The first wrong step is Step _____ . What should it say?

_____

🔍 Spot the Mistake

Volume of a cone-on-hemisphere toy

Step 1. The toy is a cone of height 2 cm on a hemisphere of radius 2 cm.

Step 2. Volume = volume of cylinder of the same radius and height − volume of cone.

Step 3. So volume = $\pi r^2 h - (1/3)\pi r^2 h$.

_____ The first wrong step is Step _____ . What should it say?

_____

🔍 Spot the Mistake

Finding the exposed area of a cube with a hemispherical depression

Step 1. A hemisphere of radius r is scooped out of one face of a cube of edge a.

Step 2. Surface area of remaining solid = TSA of cube + CSA of hemisphere.

Step 3. So area = $6a^2 + 2\pi r^2$.

_____ The first wrong step is Step _____ . What should it say?

_____

Part 10 · Quick-Fire Round

Match each solid to its volume formula

Draw a line from each item on the left to its match on the right.

#	Left	Right (shuffled)
1	Cuboid (l, b, h)	$V=\frac{2}{3}\pi r^3$
2	Cube (edge a)	$V=\pi r^2 h$
3	Cylinder (r, h)	$V=\frac{4}{3}\pi r^3$
4	Cone (r, h)	$V=l b h$
5	Sphere (r)	$V=\frac{1}{3}\pi h(r_1^2+r_1 r_2+r_2^2)$
6	Hemisphere (r)	$V=a^3$
7	Frustum (r_1, r_2, h)	$V=\frac{1}{3}\pi r^2 h$

True or False Sprint

Read each statement to yourself (or take turns with a partner). Decide True or False before you check the solutions.

#	Statement	Your call
1	The total surface area of a solid made of two basic solids equals the sum of their two separate total surface areas.	☐ True ☐ False
2	The volume of a solid made of two basic solids equals the sum of their two separate volumes.	☐ True ☐ False
3	The slant height of a cone is always greater than its vertical height.	☐ True ☐ False
4	A hemisphere has a curved surface area of $2\pi r^2$.	☐ True ☐ False
5	When a hemisphere is scooped out of a cube's face, the cube's exposed surface area decreases.	☐ True ☐ False
6	The capacity of a container can be less than the volume you would compute by ignoring an internal obstruction.	☐ True ☐ False
7	CSA stands for "Complete Surface Area".	☐ True ☐ False

#	Statement	Your call
8	For a cone, $l^2 = r^2 + h^2$.	☐ True ☐ False
9	If a solid is circumscribed exactly by a cylinder of the same radius, the cylinder's volume is always less than the solid's volume.	☐ True ☐ False
10	Lead shots dropped into a full conical vessel of water cause water to overflow equal to their own total volume (until the vessel is no longer full).	☐ True ☐ False

Two Truths and a Lie

Two statements are true, one is false. Circle the lie — then correct it.

Set 1

(a) The total surface area of a cone-on-hemisphere toy is the sum of their two curved surface areas.

(b) The total surface area of a cone-on-hemisphere toy is the sum of their two total surface areas.

(c) The volume of a cone-on-hemisphere toy is the sum of their two volumes.

The lie is _____ . Corrected: _____

Set 2

(a) A hemisphere's curved surface area is $2\pi r^2$.

(b) A hemisphere's total surface area is $3\pi r^2$.

(c) A hemisphere's total surface area is $2\pi r^2$.

The lie is _____ . Corrected: _____

Set 3

(a) Scooping a hemispherical depression out of a cube face adds πr^2 net to the cube's exposed surface area.

(b) Scooping a hemispherical depression out of a cube face subtracts πr^2 net from the cube's exposed surface area.

(c) A cone's slant height satisfies $l = \sqrt{r^2 + h^2}$.

The lie is _____ . Corrected: _____

Part 11 · The Escape Challenge

Five gates, each a calculation. Each gives you one digit. Collect all five to unlock the exit code.

🔒 GATE 1 — The Hemisphere Gate

Find the CSA of a hemisphere of radius 7 cm, using $\pi = \frac{22}{7}$. Take the first two digits of your answer.

Digit: ☐

🔒 GATE 2 — The Slant Gate

A cone has radius 3 cm and height 4 cm. Find its slant height.

Digit: ☐

🔒 GATE 3 — The Cube Gate

A cube has edge 5 cm. Find its total surface area, then take the first digit.

Digit: ☐

🔒 GATE 4 — The Toy Gate

In the classic Example 7 toy (hemisphere + cone, radius 2 cm, cone height 2 cm, $\pi = 3.14$), the volume of the toy and the "leftover" volume in its circumscribing cylinder turn out equal. Take the ones digit of that common volume, rounded to the nearest whole number (25.12 rounds to 25).

Digit: ☐

🔒 GATE 5 — The Combination Gate

How many different pairs of basic solids are named in the Learn First notes as classic combinations (cone+hemisphere, cube+hemisphere, cylinder+cone, cylinder+hemisphere)? Count them.

Digit: ☐

EXIT CODE: ☐ ☐ ☐ ☐ ☐

Formula Sheet

Cut this page out and keep it. Everything in this chapter, on one page.

CSA of cylinder

$$2\pi rh$$

TSA of cylinder

$$2\pi r(h+r)$$

CSA of cone

$$\pi rl,\ l=\sqrt{r^2+h^2}$$

TSA of cone

$$\pi r(l+r)$$

CSA and TSA of sphere

$$4\pi r^2$$

CSA of hemisphere

$$2\pi r^2$$

TSA of hemisphere

$$3\pi r^2$$

Volume of cylinder

$$\pi r^2 h$$

Volume of cone

$$\frac{1}{3}\pi r^2 h$$

Volume of sphere

$$\frac{4}{3}\pi r^3$$

Volume of hemisphere

$$\frac{2}{3}\pi r^3$$

Volume of frustum

$$\frac{1}{3}\pi h(r_1^{\,2}+r_1 r_2+r_2^{\,2})$$

Combined-solid rule (surface area)

$TSA = \sum$ *exposed curved/flat surfaces only, never the two separate TSAs*

Combined-solid rule (volume)

$$V_{combined}=\sum V_{parts}$$

Full Solutions

Not just answers — working. Compare your method with this one; if they differ but both reach the right answer, make sure you understand why yours also works.

Part 3 · Guided Practice

1. r = 7, h = 6, area = 2×(22/7)×7×13 = 572 cm² · **2.** diameter = 7, r = 3.5; 6×49=294, πr²=38.5; total = 332.5 cm² · **3.** cylinder = π(60²)(180); cone = (1/3)π(60²)(120); hemisphere = (2/3)π(60³); left = 2,036,571.43 cm³ (using π=22/7, approx.)

Practice Set A

1. Each cube has edge 4 cm (since $4^3=64$). Joined end to end: cuboid $8\times4\times4$. Surface area $¿2(lb+bh+hl)=2(32+16+32)=160\,cm^2$.

2. $r=3.5$ cm; height of cone $¿15.5-3.5=12$ cm; $l=\sqrt{3.5^2+12^2}=12.5$ cm. TSA $¿\pi rl+2\pi r^2=\frac{22}{7}(3.5)(12.5)+2\frac{22}{7}¿$.

3. Remaining surface area $¿6a^2-\pi¿$.

4. $r=1.25$ mm, cylinder length $¿14-2(1.25)=11.5$ mm. Surface area $¿2\pi r h_{cyl}+4\pi r^2$ (two hemisphere CSAs = one sphere CSA)

$¿2\frac{22}{7}(1.25)(11.5)+4\frac{22}{7}¿$.

5. $r=h=1$. Volume $¿\frac{2}{3}\pi r^3+\frac{1}{3}\pi r^2 h=\frac{2}{3}\pi+\frac{1}{3}\pi=\pi\,cm^3$.

6. $r=0.7$, $h=2.4$; $l=\sqrt{0.7^2+2.4^2}=2.5$ cm. Remaining TSA = CSA of cylinder + 2 base circles of cylinder − base circle replaced by cone base + CSA of cone $¿2\pi rh+\pi r^2+\pi rl\approx10.56+1.54+5.5\approx17.6\,cm^2\approx18\,cm^2$.

7. $r=3.5$, cylinder height between hemispheres $¿10-2(3.5)=3$ cm. TSA = CSA of cylinder + 2 × CSA of hemisphere $¿2\pi rh+4\pi r^2=2\frac{22}{7}(3.5)(3)+4\frac{22}{7}¿$.

Practice Set B

1. Cone: $r=2$, $l=2.8$; CSA $¿\pi rl=\frac{22}{7}(2)(2.8)=17.6\,m^2$. Cylinder CSA $¿2\pi rh=\frac{22}{7}(2)(2)(2.1)=26.4\,m^2$. Canvas $¿17.6+26.4=44\,m^2$; cost $¿44\times500=₹22,000$.

2. $r=1.5$, cylinder length $¿12-2(2)=8$ cm. Volume $¿\pi r^2 h_{cyl}+2(\frac{1}{3}\pi r^2 h_{cone})=\frac{22}{7}¿$.

3. $r = 1.4$, cylinder length ¿ $5 - 2(1.4) = 2.2$ cm. Volume of one gulab jamun ¿ $\pi r^2 h_{cyl} + \frac{4}{3}\pi r^3 \approx 13.55 + 11.49 \approx 25.05\,cm^3$. Syrup in 45: $45 \times 0.3 \times 25.05 \approx 338.2\,cm^3$.

4. Volume of cuboid ¿ $15 \times 10 \times 3.5 = 525\,cm^3$. Volume of 4 cones ¿ $4 \times \frac{1}{3}\pi$¿. Wood $\approx 525 - 1.47 \approx 523.53\,cm^3$.

5. Volume of cone (water) ¿ $\frac{1}{3}\pi$¿. One-fourth flows out: $\approx 52.33\,cm^3$. Each lead shot volume ¿ $\frac{4}{3}\pi$¿. Number of shots $\approx 52.33/0.524 \approx 100$.

6. Volume ¿ π¿. Mass $\approx 111,532.8 \times 8 \approx 892,262\,g \approx 892.26\,kg$.

7. Volume of spherical part ¿ $\frac{4}{3}\pi$¿. Volume of neck ¿ π¿. Total $\approx 346.5\,cm^3$, close to the measured 345 cm³ — so the child's measurement is approximately correct.

Practice Set C

1. Volume of cone ¿ $\frac{1}{3}\pi$¿. One-fourth flows out when shots are dropped: volume of water displaced ¿ $\frac{1}{4} \times \frac{200\pi}{3} = \frac{50\pi}{3} \approx 52.33\,cm^3$. Each shot ¿ $\frac{4}{3}\pi$¿. Number of shots ≈ 100.

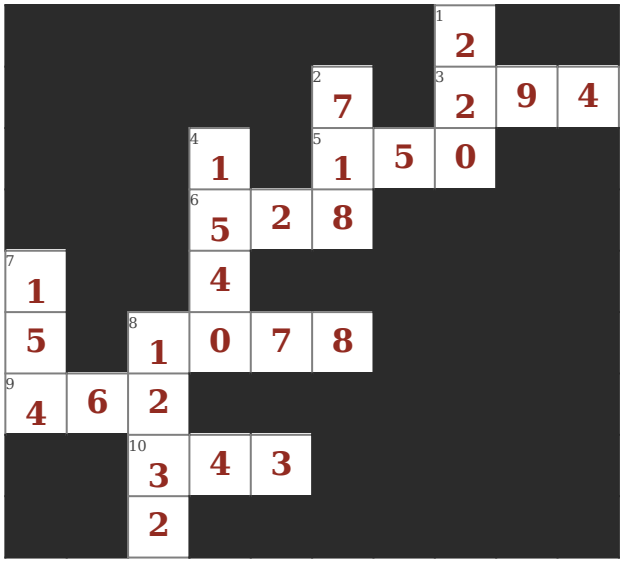
2. Apparent capacity ¿ π¿. Actual capacity ¿ $196.25 - \frac{2}{3}\pi$¿. Percentage lost ¿ $\frac{32.71}{196.25} \times 100 \approx 16.67\,\%$.

3. Volume of toy ¿ $\frac{2}{3}\pi$¿. Circumscribing cylinder: $r = 3$, $h = 3 + 4 = 7$; volume ¿ π¿. Difference ¿ $197.82 - 94.2 = 103.62\,cm^3$.

4. The two circular faces where the solids meet (e.g. the flat face of the cone and the flat face of the hemisphere in the playing top) are hidden inside the combined solid and are not part of its outer skin, so they must be left out of the total. Example: a hemisphere (TSA $3\pi r^2$) and a cone (TSA $\pi r(l + r)$) joined lose one πr^2 circle from each, so the true combined TSA is $2\pi r^2 + \pi r l$, strictly less than $3\pi r^2 + \pi r(l + r)$.

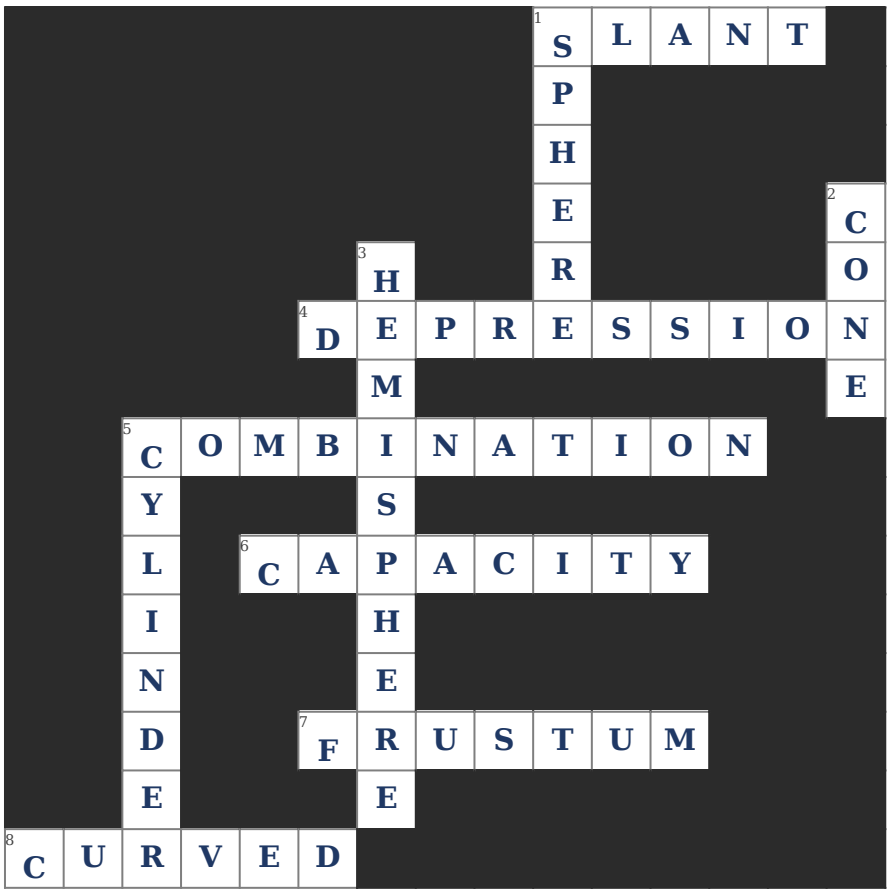
5. Volume of half-cylinder roof ¿ $\frac{1}{2}\pi$¿. A full hemisphere-topped cylinder roof of the same base circle (radius 3.5 m) would have volume $\frac{2}{3}\pi$¿ — but this is a single hemisphere over a point, not a half-cylinder over the whole 15 m ridge, so it is actually far *less* air, a decrease of about $288.75 - 89.8 \approx 198.95\,m^3$, showing the half-cylinder captures much more air along the shed's length.

Part 7 · Crossnumber — Solved Grid



Across: 3 = 294 · 5 = 150 · 6 = 528 · 8 = 1078 · 9 = 462 · 10 = 343
Down: 1 = 220 · 2 = 718 · 4 = 1540 · 7 = 154 · 8 = 1232

Part 7b · Crossword — Solved Grid



Across: 1 = SLANT · 4 = DEPRESSION · 5 = COMBINATION · 6 = CAPACITY · 7 = FRUSTUM · 8 = CURVED

Down: 1 = SPHERE · 2 = CONE · 3 = HEMISPHERE · 5 = CYLINDER

Part 8 · Number Search — Solved Grid

5	5	7	0	4	4	9	6	3	9	4	3	2
6	9	0	9	9	9	3	0	2	3	2	**2**	1
1	5	**8**	7	7	4	3	5	9	2	5	**6**	2
6	6	**7**	8	6	5	6	7	0	5	8	**4**	8
1	1	**0**	0	7	1	3	8	5	5	**5**	**2**	**8**
2	6	**1**	1	4	4	1	6	5	7	**4**	9	5
0	6	**0**	**5**	**1**	1	7	4	6	**5**	6	3	9
2	2	**0**	8	**1**	0	6	5	**1**	6	7	5	2
0	**2**	0	2	**5**	4	1	4	9	3	4	0	2
2	9	4	1	**4**	7	8	4	6	5	4	8	4
3	**4**	**3**	2	**0**	8	6	8	4	7	2	5	6
7	8	0	3	5	0	6	2	6	5	9	2	6
6	5	2	9	6	**2**	**9**	**4**	8	8	8	5	9

Part 8 · Magic Square — Solved (constant 15)

8	**1**	**6**
3	**5**	**7**
4	**9**	**2**

Part 9 · Spot the Mistake

Problem	First wrong step	It should say
Surface area of a playing top (cone + hemisphere)	Step 1	The two flat circular faces disappear into the join. Correct rule: TSA of top = CSA of cone (πrl) + CSA of hemisphere ($2\pi r^2$) — never the two total surface areas, which would double-count the vanished circle.

Problem	First wrong step	It should say
Volume of a cone-on-hemisphere toy	Step 1	This toy is a cone **plus** a hemisphere, not a cylinder minus a cone. Correct rule: Volume = (2/3)πr³ (hemisphere) + (1/3)πr²h (cone), which add because no material is shared between the two pieces.
Finding the exposed area of a cube with a hemispherical depression	Step 1	Cutting a hemisphere *out* of a face removes a flat circle of area πr² from the cube's surface, but exposes the hemisphere's curved inner surface 2πr². Correct rule: area = 6a² − πr² + 2πr² = 6a² + πr² (same net effect as adding a hemisphere, because a concave dent and a convex bump both trade a flat circle for a curved one).

Part 10 · Matching

#	Left	Correct match
1	Cuboid (l, b, h)	$V = l\,b\,h$
2	Cube (edge a)	$V = a^3$
3	Cylinder (r, h)	$V = \pi r^2 h$
4	Cone (r, h)	$V = \frac{1}{3}\pi r^2 h$
5	Sphere (r)	$V = \frac{4}{3}\pi r^3$
6	Hemisphere (r)	$V = \frac{2}{3}\pi r^3$
7	Frustum (r_1, r_2, h)	$V = \frac{1}{3}\pi h\left(r_1^2 + r_1 r_2 + r_2^2\right)$

Part 10 · True or False

1-F · 2-T · 3-T · 4-T · 5-F · 6-T · 7-F · 8-T · 9-F · 10-T

Part 10 · Two Truths and a Lie

Set	The lie	Correction
1	b	The flat circular faces where the cone and hemisphere meet are hidden inside the solid, so they must NOT be counted — only the curved surfaces survive as the outer skin.
2	c	Total surface area of a hemisphere includes the flat circular base too: $2\pi r^2 + \pi r^2 = 3\pi r^2$, not $2\pi r^2$.
3	b	The depression removes a flat circle (πr^2) but exposes a curved bowl ($2\pi r^2$) — a net gain of πr^2, not a loss.

Part 11 · The Escape Challenge

EXIT CODE: 15 · 5 · 1 · 5 · 4

(Answer: 15, since CSA = 154 cm²) (Answer: 5) (Answer: 1, since TSA = 150 cm²) (Answer: 5) (Answer: 4)

From a playing top to a bird-bath, every combined solid this chapter throws at you reduces to the same two golden rules: surface areas only count what is still exposed, volumes simply add. Get those two sentences right and the arithmetic takes care of itself. □□